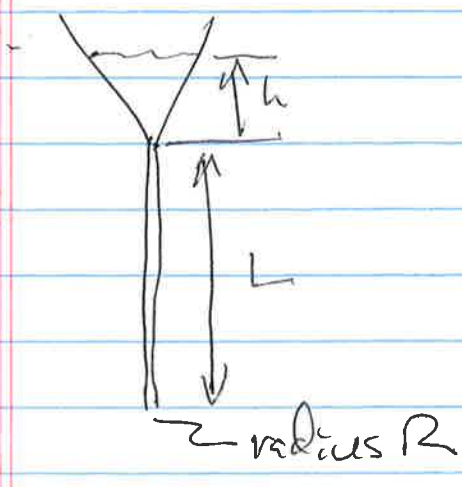
How to turn a funnel into a viscometer



Because the flow rate for Poiseuille's Law goes as R^4, all (or almost all) of the ΔP occurs in the stem of the funnel

Thus: $$\frac{\partial P}{\partial z} \cong -\frac{\rho g(L+h)}{L} = -\rho g\left(1+\frac{h}{L}\right)$$

The flow rate Q is:

$$Q = -\frac{\partial P}{\partial z}\frac{\pi}{8}\frac{R^4}{\mu} = \frac{\pi}{8}\frac{R^4}{\mu}\rho g\left(1+\frac{h}{L}\right)$$

We need an equation for the drainage time

~~Most~~ The volume of the funnel is

$$V = \pi r^2 \frac{h}{3}$$

but $r = h\tan\frac{\theta}{2}$ where θ is the cone angle

For a 60° cone angle $r = \frac{h}{\sqrt{3}}$

and $V = \frac{\pi}{9} h^3$

If we put in a volume V_0 the initial height h_0 is:

$$h_0 = \left(\frac{9 V_0}{\pi}\right)^{1/3}$$

The flow rate is the change in vol V:

$$Q = -\frac{\partial V}{\partial t} = -\frac{\pi}{3} h^2 \frac{\partial h}{\partial t} = \frac{\pi}{8} \frac{R^4}{\mu} \rho g \left(1 + \frac{h}{L}\right)$$

So $$\frac{\partial h}{\partial t} = -\frac{3}{8} \frac{R^4}{\mu} \frac{\rho g}{h^2} \left(1 + \frac{h}{L}\right)$$

Let's scale: $h^* = h/h_0$, $t^* = t/t_c$

$\lambda \equiv h_0/L$ (a parameter)

$$\therefore \frac{h_0}{t_c} \frac{\partial h^*}{\partial t^*} = -\frac{3}{8} \frac{R^4}{\mu} \frac{\rho g}{h_0^2} \frac{1}{h^{*2}} \left(1 + \lambda h^*\right)$$

Dividing out:

$$\frac{\partial h^*}{\partial t^*} = -\left[\frac{3}{8} \frac{R^4}{\mu} \frac{\rho g t_c}{h_0^3}\right] \frac{1 + \lambda h^*}{h^{*2}}$$

we take the term in brackets to be unity!

$$\therefore t_c = \frac{8}{3}\frac{\mu h_o^3}{R^4 \rho g}$$

and $$\frac{dh^*}{dt^*} = -\frac{1+\lambda h^*}{h^{*2}} \qquad h^*\Big|_{t^*=0} = 1$$

Rearranging:

$$\frac{h^{*2}}{1+\lambda h^*}\, dh^* = -dt^*$$

The drainage time $t_d^* = t_d/t_c$ is:

$$t_d^* = \int_0^1 \frac{h^{*2}}{1+\lambda h^*}\, dh^*$$

for $\lambda \ll 1$ we would get $t_d^*\Big|_{\lambda\to 0} = \frac{1}{3}$

for finite λ we get:

$$t_d^* = \frac{\lambda(\lambda-2) + 2\ln(1+\lambda)}{2\lambda^3}$$

from wolfram alpha...

Suppose we put in a volume V_0 into the cone of the funnel. This yields

$$\lambda = \frac{h_0}{L} = \frac{1}{L}\left(\frac{9V_0}{\pi}\right)^{1/3}$$

and some t_d^*

If we <u>measure</u> t_d then

$$t_d = t_d^* t_c = t_d^* \frac{8}{3} \frac{\mu h_0^3}{R^4 \rho g}$$

$$\text{so } \mu = \frac{3}{8} \frac{t_d R^4 \rho g}{t_d^* h_0^3}$$

$$h_0^3 = \frac{9}{\pi} V_0$$

$$\text{so } \mu = \frac{\pi}{24} \frac{t_d R^4 \rho g}{t_d^* V_0}$$

~~thus~~ We can also look at $\nu \equiv \frac{\mu}{\rho}$:

$$\nu = \frac{\pi}{24} \frac{t_d R^4 g}{t_d^* V_0}$$

Let's try this! The funnel has a stem length $L = 14\,cm$

By filling the stem w/ water & draining we measure a vol. of 1.88 ml

This yields $R = \left(\frac{1.88}{14\pi}\right)^{1/2} = 0.207\,cm$

We fill the stem & add 25 ml extra

So $V_0 = 25\,cm^3$, $h_0 = 4.153\,cm$

$$\lambda = \frac{h_0}{L} = 0.30$$

$$t_d^* = 0.273$$

$$\text{so } \nu = \frac{\pi}{24}\,\frac{t_d\,(0.207)^4(980)}{(0.273)(25)}$$

$$= 0.0345 * t_d$$ (stokes where t_d is in sec)

so a 1 stoke fluid should drain in 29 s

A 90% by volume glycerin/water sol'n has a kinematic visc. of 2 stokes, so it should take about 58 s to drain.

This funnel only works for viscous fluids. What if we use water?

$\nu_{water} = 0.01$ stokes (1 cs)

so it should drain in ~ 0.3 s!

If you do it you find it is more like 3 s!

Why? We didn't include inertia!

You have to accelerate the fluid.

Ignoring viscosity entirely we have

$$\frac{1}{2} \rho U^2 = \rho g \Delta H \quad \text{(Bernoulli's eq'n)}$$

$$\therefore \quad Q = \pi R^2 (2 g \Delta H)^{1/2}$$

$$t_\Delta = \frac{V_0}{Q} = \frac{V_0}{\pi R^2 (2g\Delta H)^{1/2}} \approx 1 s$$

which is much longer than the viscous term!

In addition, the Re is quite high - it may be turbulent causing further losses!